RHEODYNAMICS OF A DISK ROTATING IN A NON-NEWTONIAN LIQUID

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The power approximation to the consistency flow curve of a rotating disk is considered; a criterial relationship for the rheodynamic resistance of the disk is then deduced. Experimental data obtained in solutions of sodium carboxymethyl cellulose (CMC) for C = 0–1.5% agree closely with the proposed generalization.

The practical use of the Tomes effect in open or closed hydraulic systems [1-3] requires the derivation of a criterial relationship for the rheodynamic resistance of a rotating disk, the latter being an inherent part of any bladed machine.

When studying the rheodynamic resistance of disks rotating in a non-Newtonian liquid it is desirable to use the power approximation to the consistency flow curve [4] for a rotating disk,

$$P = k'' V^{n''}. \tag{1}$$

Using the Cochran solution [5] we may write the integrated mean values of the stress and shear strain rate in the form

$$P = \frac{4}{3} \cdot \frac{M}{R^3}; \quad V = 1.29\, \omega \sqrt{\mathrm{Re}}. \tag{2}$$

The rheodynamic resistance coefficient of a rotating disk wetted on both sides is determined, after allowing for Eqs. (1) and (2), by

$$C_M \equiv \frac{2M}{\frac{1}{2}\rho\omega^2 R^5} = \frac{3^{1-n''} A^{n''}}{\mathrm{Re}'''^{0.5}}. \tag{3}$$

The generalized Reynolds number Re''' in Eq. (3) is described in the following way:

$$\mathrm{Re}''' = \frac{\mathrm{Re}''^2}{\mathrm{Re}^{n''}}; \tag{4}$$

$$\mathrm{Re}'' = \frac{\rho R^2 \omega^{2-n''}}{k''}. \tag{5}$$

Some preliminary results of an experimental verification of the proposed generalization of the rheodynamic resistance of a rotating disk were presented in [6]. In the present investigation a detailed experimental verification of Eq. (3) was carried out in a "smooth disk — shell" system filled with aqueous solutions of sodium carboxymethyl cellulose (CMC) with a mass concentration of C = 0–1.5% at t = 25°C. The experiments in the "disk-in-shell" apparatus embraced the following ranges of characteristic geometrical parameters: disk diameters d = 120–128.5 mm with a relative width of b/R = 0.025–0.055, relative axial and radial gaps, respectively, S/R = 0.0659–0.487 and a/R = 0.011–0.083. The casing or shell of the apparatus limiting the space around the rotating disk was immersed in a U10 thermostat which thermally stabilized the liquid under test. The mechanical drive was a dc motor providing a smooth variation between n = 5 and 130 rps. The torque developed by the disk was measured with a thin-walled (hollow) tensometric shaft and a mercury amalgam current take-off. The error in measuring the torque and the rate of revolution was no greater than 1% [7, 8].

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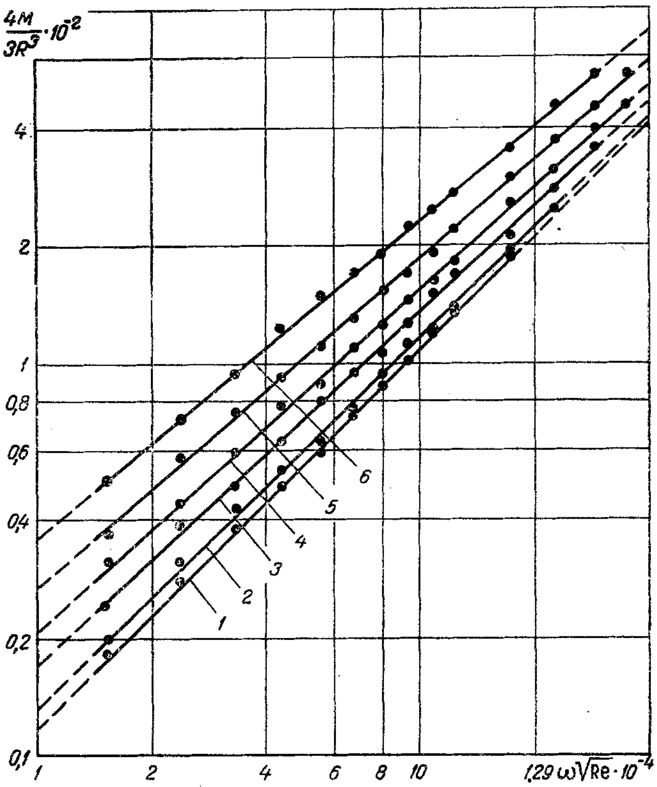

Fig. 1. Flow curves of CMC solutions in the logarithmic anamorphosis: 1) C = 0.1%; 2) 0.25; 3) 0.5; 4) 0.75; 5) 1.0; 6) 1.5. $(4M/3R^3)\cdot 10^{-2}$, N/m²; $\omega\sqrt{\mathrm{Re}}\cdot 10^{-4}$, sec⁻¹.

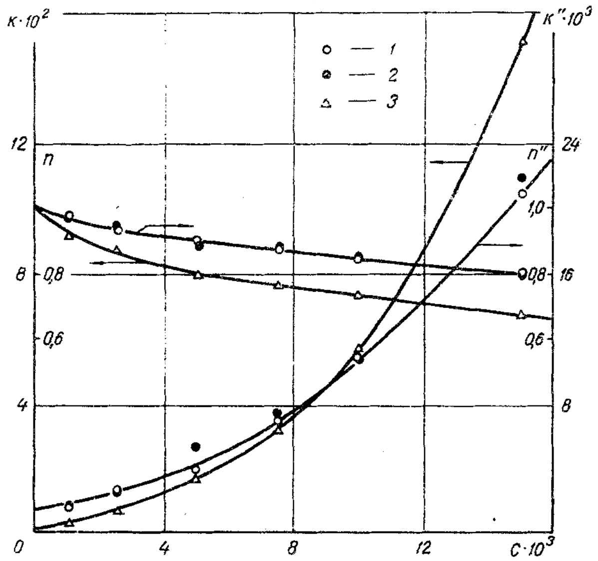

Fig. 2. Rheological parameters as functions of the mass concentration of the CMC solutions: 1) approximation of the consistency flow curves by reference to experimental data for a rotating disk; 2) correlation based on Eqs. (10) and (11); 3) based on viscometric data obtained by the capillary method. $k\cdot 10^2$, N·sec/m²; $\kappa''\cdot 10^3$, $N\cdot sec^{n''}/m^2$.

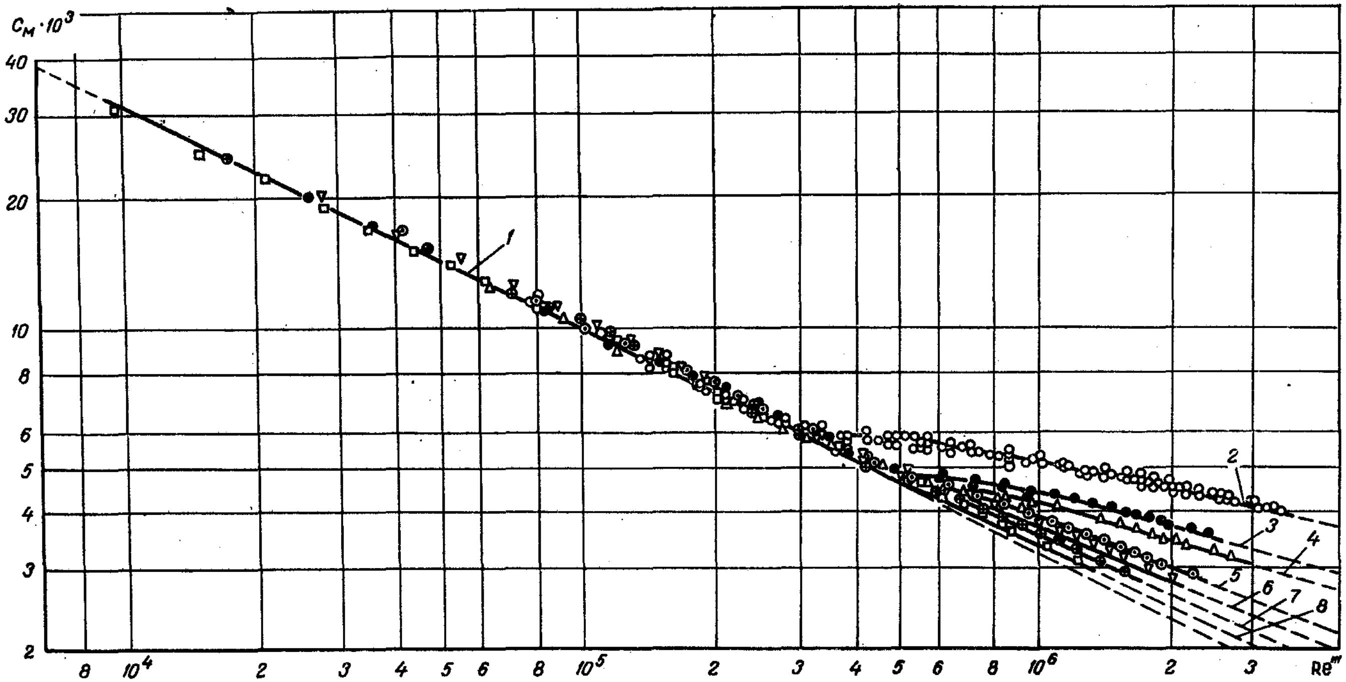

Fig. 3. Moment-of-resistance coefficient of a rotating disk in a shell (S/R = 0.206) immersed in CMC solutions as a function of the generalized Reynolds number: 1) C = 0–1.5%; 2) 0; 3) 0.1; 4) 0.25; 5) 0.5; 6) 0.75; 7) 1.0; 8) 1.5.

The rheological constants n" and k" were determined by solving the power equation (1) with due allowance for (2) by reference to the experimentally measured characteristic $M = f(\omega)$ for a smooth disk 128.5 mm in diameter rotating in CMC solutions in an unbounded space (mass concentrations C = 0-1.5%). For testing the stationary liquid in free space the shell of the apparatus was taken away, and the disk, supported in cantilever fashion on a vertical axis, was immersed in a special tank, the specified temperature being maintained in this tank by the circulation of liquid through a TS-24 thermostat.

The rectilinear anamorphosis of the consistency flow curves for a rotating disk in the range of shear velocities under consideration (Fig. 1) enables us to derive a power approximation with a mean-square error as low as 1% and to obtain satisfactory correlation for n" and k" with respect to the concentration of the CMC solutions (Fig. 2).

Thus, the experimental data presented here indicate the absence of any marked change in the mode of secondary flow induced by the disk; the fact that the experimental range of measurement of the $M = f(\omega)$ characteristic lies close to Re_*''' indicates that centrifugal flow is taking place around the rotating disk during the experiments [9].

Since the ranges of variation of the axial gaps and the Reynolds numbers ($Re = 9 \cdot 10^4 - 4 \cdot 10^6$) studied satisfy the conditions characteristic of the existence of separate boundary layers on the stator and rotor [10] (this mode of flow being realized, for example, in centrifugal pumps [11]), Eq. (3) enables us to generalize the rheodynamic resistance of a disk rotating in a shell (Fig. 3).

For the range of CMC concentrations and the characteristic geometrical parameters of the disk — shell system studied we obtain a unique value of the critical Reynolds number,

$$Re_*''' \equiv \frac{Re''^2}{Re^{n''}} \simeq (3-5) \cdot 10^5. \tag{4'}$$

For $Re''' > Re_*'''$ the uniqueness of Eq. (3) is disrupted, and it is converted into a family of straight lines with a parameter n" qualitatively repeating the well-known generalization of Metzner and Reed for tubes [4].

For the laminar and turbulent modes of flow around a rotating disk in a shell we obtain an approximation for the experimental data (based on ~1200 experimental points) with mean-square errors of ~1.5 and ~6%, respectively.

For $Re''' < 3 \cdot 10^5$, S/R = 0.066-0.49:

$$C_M = \frac{3^{1-n''}\{1.97\,[\lg(S/R + 0.224) + 2.04]\}^{n''}}{Re'''^{0.5}}, \tag{6}$$

for $Re''' > 7 \cdot 10^5$, S/R = 0.066-0.49:

$$C_M = \frac{0.338 \exp 19\,(1 - n'')\,[\lg(S/R + 2.37) - 0.165]}{Re'''^{0.2+1.55(1-n'')}}. \tag{7}$$

These experiments once more demonstrate the unusual ability of CMC solutions (already described in [12]) to suppress turbulent pulsations. We see from Fig. 3 that even for C = 1.5%, the $C_M = f(Re''')$ curve for $Re''' > Re_*'''$ very closely approaches the limiting asymptote corresponding to laminar flow around the object. Hence, for C > 1.5% (n" < 0.81), $Re''' > Re_*'''$ and S/R = 0.066-0.49 the resistance coefficient C_M should be determined from Eq. (6), and not Eq. (7), which was obtained as a result of a generalization of the experimental data in the range C = 0-1.5% (n" = 1-0.81).

For n" = 1 and $k'' \equiv \mu$, Eq. (3) transforms into the ordinary relationship for a Newtonian liquid in the case of laminar flow around a rotating disk [5], while (6) and (7) transform into the corresponding equations describing the change in the moment of the resistance coefficient of the disk as a function of the Reynolds number and the axial gap [13].

It follows from Eqs. (6) and (7) that a relative change in the axial gap S/R has a similar effect on C_M for both Newtonian and non-Newtonian liquids. This is apparently a consequence of the retention of the separate boundary layers on the stator and rotor in the range of S/R values considered, even in the case of a disk rotating in a non-Newtonian liquid.

These experimental data regarding the rheodynamic resistance of a rotating disk in CMC solutions with C = 0-1.5% in an unlimited space agree closely with the solution of Mitschka, which on making allowance for [15] reduces to

$$C_M = \frac{3.87}{\mathrm{Re}_m^{0.5}}, \tag{8}$$

$$\mathrm{Re}_m = \left(\frac{\rho R^2 \omega^{2-n}}{k}\right)^{\frac{2}{1+n}} 6.13^{\frac{n-1}{n+1}}. \tag{9}$$

Using the Mitschka solution [14] and assuming that the $C_M = f(\mathrm{Re}''')$ and $C_M = \varphi(\mathrm{Re}_m)$ relationships are invariant with respect to ρ, R, and ω, we may establish a relationship between the rheological parameters n", k" and n, k. It thus follows from (3), (4), (5), and (8), (9) that

$$n'' = \frac{2n}{1+n}, \tag{10}$$

$$k'' = k\left(\frac{\mu}{k}\right)^{\frac{n}{1+n}} 10.2^{\frac{1-n}{2(1+n)}}. \tag{11}$$

The values of n" and k" calculated from Eqs. (10) and (11) agree closely with direct measurement (Fig. 2).

We have thus established the following.

1. The generalized Reynolds number (4) is a criterion of rheodynamic similarity, and for the flow of a non-Newtonian liquid around a rotating disk uniquely establishes a crisis at $\mathrm{Re}_*''' \simeq (3\text{-}5)\cdot 10^5$.

2. The rheodynamic resistance coefficients of disks rotating both in the free space of a non-Newtonian liquid and in a shell (casing) with S/R = 0.066–0.49, a/R = 0.01–0.08, and b/R = 0.02–0.06 may be calculated from the corresponding equations (3), (6), and (7) using the values of n", k" or n, k (n', k'), determined by viscometric measurements on solutions of high polymers.

NOTATION

$P = f(\tau)$; $V = f(\dot{\gamma})$	are the generalized variables of the consistency flow equations for a rotating disk;
τ	is the shear stress at the wall, N/m^2;
$\dot{\gamma}$	is the shear strain rate, sec^{-1};
n, n', n" and k, k', k"	are the rheological parameters determined for parts of the flow curves which are linear in the logarithmic anamorphosis;
R	is the disk radius, m;
ω	is the angular frequency of rotation, sec^{-1};
M	is the moment of resistance of the rotating disk wetted from one side, N·m;
$\mathrm{Re} = \rho\omega R^2 \mu^{-1}$	is the Reynolds number;
μ	is the viscosity, $N\cdot sec/m^2$;
C_M	is the rheodynamic resistance coefficient of the rotating disk;
A	is the coefficient in the equation for determining the hydrodynamic resistance of a rotating disk in a Newtonian liquid (A = 3.87 for a disk rotating in the free space of a stationary liquid);
Re", Re"'	are the generalized Reynolds numbers determined by Eqs. (4) and (5);
C	is the mass concentration;
t	is the temperature, °C;
d	is the disk diameter, m;
b	is the disk width (thickness), m;
S and a	are the axial and radial gaps between the disk and the shell walls, respectively, m;
n	is the number of revolutions, rps;
Re_*'''	is the critical Reynolds number;
Re_m	is the generalized Reynolds number derived from Eq. (9).

LITERATURE CITED

1. I. T. Él'perin, L. I. Levental', and Yu. N. Chesnokov, Teploénergetika, No. 10, 20 (1971).
2. G. P. Verkhivker, T. A. Rogovskii, and Yu. A. Dudin, Teploénergetika, No. 1, 85 (1972).
3. A. G. Fabula, Trans. ASME, Ser. D, J. Basic Eng., 93, No. 3, 109 (1971).
4. A. B. Metzner and J. G. Reed, AChE J., 1, No. 4, 434 (1955).
5. W. G. Cochran, Proc. Camb. Phil. Soc., 30, No. 3, 365 (1934).

6. Z. R. Gorbis and T. A. Rogovskii, in: Heat and Mass Transfer, [in Russian], Vol. 3, Minsk (1972).
7. G. P. Verkhivker, T. A. Rogovskii, and Yu. A. Dudin, Izv. Vyssh. Ucheb. Zaved., Énergetika, No. 10, 145 (1972).
8. T. A. Rogovskii and V. M. Mel'nik, in: Cybernetics and Automatic Control [in Russian], Kiev (1973).
9. G. S. Hansford and M. Litt, Chem. Eng. Sci., 23, 849 (1968).
10. J. W. Daily and R. E. Nece, Trans. ASME, Ser. D, 82, No. 1, 217 (1960).
11. A. A. Lomakin, Centrifugal and Axial Pumps [in Russian], Mashinostroenie, Moscow — Leningrad (1966).
12. A. V. Lykov, Z. P. Shul'man, and B. I. Puris, Inzh.-Fiz. Zh., 14, No. 1, 17 (1968).
13. T. A. Rogovskii and Z. R. Gorbis, "Hydrodynamic resistance of the disks on the wheel of a centrifugal pump," Teploénergetika, No. 9, 46 (1973).
14. P. Mitschka and J. Ulbrecht, J. Appl. Sci. Res., A15, 345 (1966).
15. K. Wichterle and J. Ulbrecht, Rheologica Acta, 6, No. 4 (1967).